Stability of connected cylindrical liquid bridges

Neha M. Patel,[1] Mohammad Reza Dodge,[1] J. Iwan D. Alexander,[1,2] Lev A. Slobozhanin,[2] P. L. Taylor,[1] and Charles Rosenblatt[1]

[1]*Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106*

[2]*Department of Mechanical and Aerospace Engineering, Case Western Reserve University, Cleveland, Ohio 44106*

(Received 15 May 2001; published 18 January 2002)

Two cylindrical liquid bridges, with a conduit to facilitate flow of liquid from one bridge to the other, were levitated against gravity in a magnetic field gradient. The stability limit of the bridges subjected to near zero total body force was measured as a function of their slenderness ratios, and found to be in good agreement with theoretical predictions.

DOI: 10.1103/PhysRevE.65.026306 PACS number(s): 62.10.+s, 81.70.Ha, 75.50.Mm

Liquid bridges, which are regions of liquid supported by two or more solid surfaces, have been attracting scientific attention since the time of Plateau and Rayleigh [1,2]. The stability and ultimate collapse of such bridges is of interest because of their importance in industrial processes such as zone refining, and their potential for low gravity applications. For a weightless cylindrical bridge of length L and diameter d (supported by two circular disks), it was shown theoretically [1–3] and experimentally [1,4] that the maximum stable slenderness ratio Λ [$\equiv L/d$] is π. When subjected to gravity, the cylindrical shape of a bridge tends to deform, limiting its stability and decreasing the maximum achievable value of Λ. Extensive investigations, both theoretical and experimental, have examined the stability of cylindrical volume bridges as a function of Bond number B [$\equiv g\rho d^2/4\sigma$], where g is the gravitational acceleration, ρ the density, and σ the surface tension. Both axial [5–10] and lateral [5,11–13] gravitational forces have been analyzed.

Nothwithstanding our extensive knowledge of isolated liquid bridges and other fluid structures, many practical problems involving disconnected free surfaces, such as liquids in porous media, have received scant attention. Examples of such systems include a pair of pendant or sessile fluid droplets or a pair of liquid bridges connected through a liquid-filled tube, where each bridge or droplet surface is referred to as a ‘‘connectivity component’’ of the free surface. Here one must consider the connectivity components, solid wall(s), and liquid region together, with the constraint that the total liquid volume be constant. Despite the fact that general solution methods and applications to specific configurations of connected domains have been discussed theoretically in the literature [8,14–17], there are no extant experimental data to our knowledge, largely owing to the difficulty in maintaining zero total body force in a non-space-borne environment. To address this deficiency we examined experimentally the problem of two weightless cylindrical liquid bridges connected by a single conduit. Our approach is to use the technique of magnetic levitation [18,19], which facilitates continuous control of the total body force, and thus allows a zero body force environment.

The bridge assembly is placed in a Faraday magnet, whose pole pieces are shaped so that the quantity $H_x \partial H_x/\partial z$ is nearly uniform over a sizable volume of space, where the vertical axis z is parallel to the gravitational field and is directed upward. Figure 1(a) shows a top view and Fig. 1(b) shows an end view of the magnet and liquid bridge apparatus. A paramagnetic liquid of susceptibility χ experiences two body forces: a magnetic force per unit volume $\chi H_x \partial H_x/\partial z$ directed along the $+z$ axis, as well as a downward gravitational force per unit volume ρg. The Bond number corresponding to ‘‘effective gravity’’ must therefore be redefined as $B \equiv (g\rho - \chi H_x \partial H_x/\partial z) d^2/4\sigma$. By adjusting the magnet current so that these two forces cancel, a quasi-gravity-free environment may be obtained. We have previously used this powerful technique to examine the stability of single bridges [18], including noncylindrical volume bridges having reduced volume $V_r \neq 1$ [19]. (Note that V_r is defined as the volume of fluid divided by $\pi d^2 L/4$.) We have also performed dynamic studies of collapsing liquid bridges [20] and bridge resonances [21] by temporally varying the magnet current. The profiles of both H_x and $\partial H_x/\partial z$ for our magnet are reported in Ref. [18]. Despite the near zero total body force, small residual forces remain, which tend to

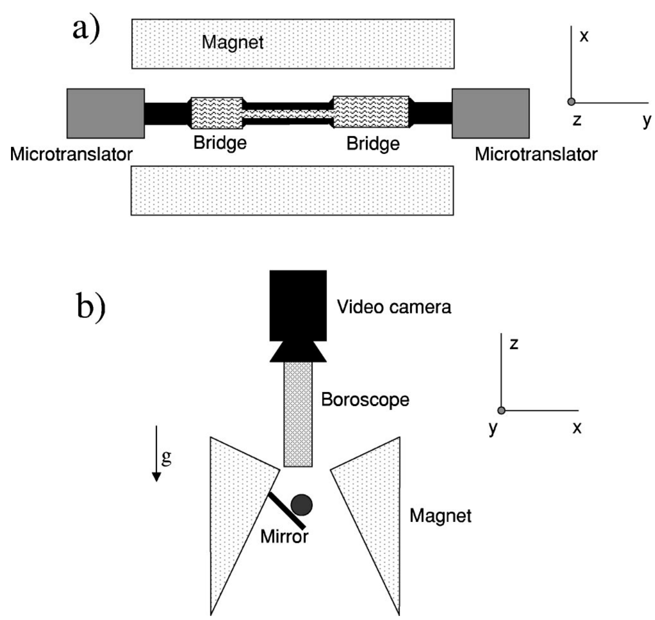

FIG. 1. Schematic view of experimental setup. (a) Top view. (b) End view. Arrow shows direction of gravity; magnetic force is equal and antiparallel to gravity.

slightly reduce the stability of the system. First, the magnetic component of force along the z axis is uniform to better than $\pm 2\%$ over a 4 mm distance—this corresponds to the diameter of the bridges (Fig. 1). Second, because the bridge is centered between the two pole pieces, by symmetry the horizontal force $\chi H_x \partial H_x / \partial x$ vanishes along the bridges' axes. Off axis, however, there is a nonzero force proportional to x that pulls the fluid toward the two pole pieces. This force is largest at the bridges' maximum value of x, and corresponds to about $0.03g$. Taken together, these residual forces tend to reduce stability, as will be discussed below.

The bridge support apparatus consists of a pair of cylindrical aluminum rods that were machined with conical tips to serve as wetting barriers [see Fig. 1(a)]. The diameter of the tips is 4.00 mm. Additionally, a short cylindrical center piece was machined with similar conical tips at both ends, as well as a 2 mm diameter axial hole to serve as a conduit for fluid between the two liquid bridges. The two rods were mounted horizontally along the y axis on a y-axis microtranslation stage, and centered midway between the magnet pole pieces. The center piece was mounted coaxially with the rods, and was fixed in position. In this manner the slenderness ratio Λ_i of each of the two bridges i ($i=1,2$) could be adjusted separately and with fine precision by adjusting the two microtranslation stages. Mirrors were affixed at an angle of 45°, as shown in Fig. 1(b). This facilitated simultaneous observation of both x-axis and z-axis deformations of the two bridges by means of a boroscope attached to a charge-coupled device camera. The image was viewed on a monitor and recorded by a video cassette recorder.

A 62.5 wt % mixture of manganese chloride tetrahydrate ($MnCl_2 \cdot 4H_2O$) in distilled water was used as the paramagnetic liquid. Previously, we determined that $\rho = 1.45 \pm 0.01\ \mathrm{g\,cm^{-3}}$, $\sigma = 65 \pm 5\ \mathrm{erg\,cm^{-2}}$, and $\chi = 5.54 \pm 0.05^{-5}$ cgs for this mixture [18,20]. The magnet current was first increased so that the upward magnetic force approximately balanced the downward gravitational force, and the two rods were translated so that the gaps for the two bridges were small, typically about 2 mm. Using a butterfly syringe and hypodermic needle, a small amount of the water/$MnCl_2 \cdot 4H_2O$ mixture was injected into the axial hole in the center piece, filling the entire conduit. More liquid was then injected, wetting the ends of the rods and filling the two gaps, thereby creating two connected liquid bridges. Note that a container of steaming water was kept below the magnet in order to saturate the air and prevent evaporation of water from the bridges. One of the gaps was then widened to a fixed separation L_1 using the microtranslation stage, and more liquid was injected so that the liquid bridge was cylindrical and of slenderness ratio Λ_1. Keeping the length of this bridge fixed, gap 2 was then opened in small steps of 0.05 mm, each opening requiring approximately 2 s to complete. Between each step liquid was injected into the gap over approximately a 5 s time period so that both bridge 1 and bridge 2 were again cylindrical. If the bridges were stable and maintained their cylindrical shape for approximately 5 min, the gap in bridge 2 was further increased by 0.05 mm, liquid was added, and the procedure was repeated until the bridge collapsed. Note that it made no difference if the liquid was injected first, and the gap was subsequently increased. We examined numerous bridge pairs whose values for Λ_1 were in the region $1.352 < \Lambda_1 < \pi$. (This region corresponds to nonisochoric critical perturbations; see the discussion below.) For a given value of Λ_1, we found that above a certain value of Λ_2 the bridges would no longer maintain their cylindrical shapes: The liquid would flow from the longer bridge to the shorter bridge irreversibly, which would result in a collapse of both bridges. This was taken as the stability limit for the pair Λ_1, Λ_2, and is shown in Fig. 2. The entire process was then repeated for a new value of Λ_1. For $\Lambda_1 < 1.352$, the breaking process was different: bridge 2 became unstable and broke for Λ_2 near π. Note that we never observed a pair of deformed but stable bridges. Rather, the bridges remained cylindrical, at least within the limits of our ability to detect shape changes, right up to the stability limit. When the bridges did begin to deform, they did so catastrophically.

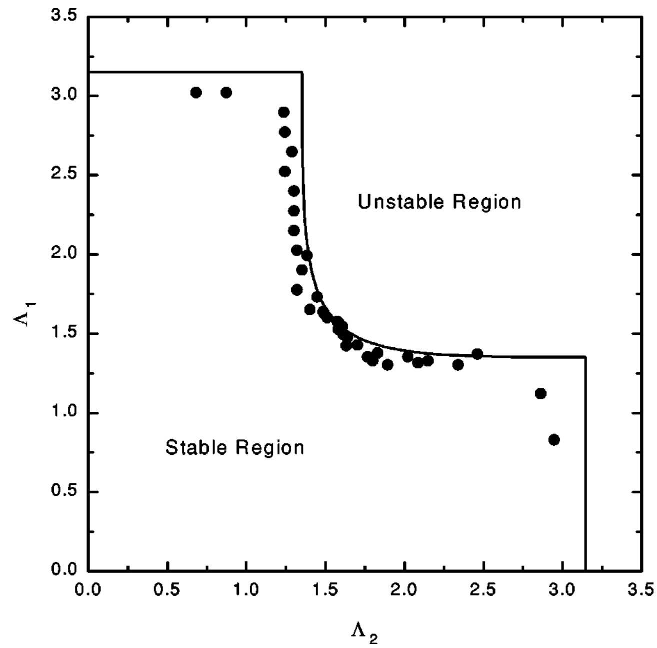

FIG. 2. Experimental data for stability limit of bridge. Solid trace is theoretical prediction.

Let us now turn to the theoretical predictions for our system. The problem of two weightless cylindrical liquid bridges connected by a single conduit was examined theoretically by Gillette and Dyson [15]. Assuming that both bridges are of diameter d and their length-to-diameter ratios are Λ_1 and Λ_2, they found that the region of stability is bounded not only by the conditions $0 < \Lambda_1 < \pi$ and $0 < \Lambda_2 < \pi$, but also by the inequality

$$\sum_{i=1}^{2} \frac{\cos \Lambda_i}{\sin \Lambda_i - \Lambda_i \cos \Lambda_i} > 0. \tag{1}$$

The boundary segments $\Lambda_1 = \pi$ (and $\Lambda_2 = \pi$) correspond to perturbations that are isochoric on each cylindrical surface and are critical for bridge 1 (and, respectively, for bridge 2). Along $\Lambda_1 = \pi$, the normal component of critical perturbations to the surface for component $i=1$ is $C_1 \sin(2\pi y_1/L_1)$, where y_1 is measured along the axis of symmetry from one of the disks of bridge 1. The loss of stability results in the breaking of bridge 1 into two unequal parts [22,23]. In contrast, the curve defined by Eq. (1) in the region $1.352<\Lambda_i<\pi$ $(i=1,2)$ corresponds to critical perturbations that are nonisochoric on each cylindrical surface, but satisfy the liquid volume conservation in aggregate. When losing stability, one bridge increases in volume and the other decreases. In particular, for the case of $\Lambda_1=\Lambda_2$, the critical value is $\Lambda_i=\pi/2$ $(i=1,2)$, and the critical perturbations are $C_i \sin(\pi y_i/L_i)$ $(i=1,2)$, where $C_1=-C_2$. Here, under critical perturbations, one of the bridges protrudes symmetrically about its equatorial plane, and the other narrows symmetrically. Bifurcation analysis performed in Ref. [23] for equidimensional critical bridges has shown that the point $\Lambda_1=\Lambda_2=\pi/2$ is a turning point in pressure. Finally, since the boundary curve [Eq. (1)] is a locus of turning points in pressure, there is no stable equilibrium beyond this curve.

Let us now examine the experimental data in light of theory. Figure 2 shows both the experimental data (points) and theoretical predictions (solid lines) for the stability limit of the double bridge system. The apparent scatter in the data—as opposed to discrepancies between experiment and theory—comes about for two reasons: Unavoidable convection currents in the air due to magnet heating tend to destabilize the the bridge randomly near its stability limit, and our ability to obtain a precise cylinder volume $V_r=1$ is limited by our imaging system. We estimate that convection may be responsible for scatter of approximately 2%. To understand the role of deviations from cylindricality, we estimate that V_r may differ from unity by as much as 2% or 3% for any given bridge. Since the theory of generalized double bridges, i.e., $V_r\neq 1$, is not completely developed, we will use known results from the theory of single bridges in order to estimate the effect that uncertainties in V_r have on the experimental scatter. [Note, however, that the single bridge theory is fully valid for boundary segments $\Lambda_i=\pi/2$ $(i=1,2)$.] For a single weightless bridge with V_r close to 1, the critical value Λ may be found from the relation [24]

$$V_r=1+2\left(\frac{\Lambda}{\pi}-1\right)+\frac{5}{2}\left(\frac{\Lambda}{\pi}-1\right)^2. \tag{2}$$

Consequently, the critical Λ equals 0.99π when $V_r=0.98$. Thus, the scatter in the data due to deviations from cylindricality would correspond to approximately 1–2 %.

The agreement between experiment and theory is very good, although not excellent. Over most of Λ_1, Λ_2 parameter space the measured stability of the bridges is less than the predicted values. Although the observed discrepancies would be considered moderately large for vertical bridges in an axial gravity environment, discrepancies of this size are not unusual in experiments in which the bridge axis is perpendicular to gravity [5,19]. One source for the discrepancies between experiment and theory is alignment in the magnetic field: If the magnetic force is not precisely antiparallel to the gravitational force, a nonzero axial Bond number (i.e., a very small axial component of force in the horizontal bridge) may exist in what is supposed to be a weightless enironment. Alignment precision of the pole pieces relative to the horizontal direction in the laboratory frame is limited to an angle of approximately ~ 0.002 rad; the end rods and center piece were then aligned with the magnet pole pieces. In consequence, an axial component of gravity as large as $0.002g$ may have been present, corresponding to a small but nonzero axial Bond number $B_{\mathrm{axial}}\sim 0.0015$. The asymptotic form for Λ for an isolated single bridge in the presence of a nonzero axial force is [12]

$$\Lambda=\pi\left[1-\left(\frac{3}{2}\right)^{4/3}B_{\mathrm{axial}}^{2/3}-\frac{1}{4}\pi^2 B_{\mathrm{lat}}^2\right], \tag{3}$$

where B_{lat} is the vertical component of the Bond number and, in principle, is adjusted to zero experimentally. Thus, for a single bridge at least, the maximum achievable slenderness ratio Λ would be reduced by approximately 2% in the presence of an axial gravity component having $B_{\mathrm{axial}}=0.0015$. If the same situation were applicable to the double bridge, it would account for approximately half of the discrepancy between theory and experiment in the vicinity of the straight boundary segments, where the single bridge theory is applicable. Unfortunately, the weight and design of the magnet prevent us from performing this experiment with the bridges' axes perfectly parallel to the gravitational force. Another issue is convection in the air, which comes about because of magnet heating and the necessity to maintain a humid atmosphere. We have worked diligently to minimize this problem, although we cannot completely eliminate it. In addition to convection causing scatter in the data, it also has the effect of slightly reducing the overall stability by perturbing the bridges when they are near their stability limit. The accuracy of the experiment is also limited by the nonuniformity of ∇H^2. As noted above, the total body force vanishes only along the axes of the bridges; there is a nonzero force, increasing with radius r, as one moves radially from the center of the bridges. This force is neither azimuthally symmetric nor equivalent to a nonzero Bond number (which involves a spatially uniform force). For example, along the x axis the horizontal force $\chi H_x \partial H_x/\partial x$ vanishes at the center of the bridge, but is approximately $0.03g$ at the bridge's outer surface. Although there is no extant theory for the effect of this force on the stability of the bridge, we believe that it tends to reduce the overall stability of the system. Finally, we point out that the apparent rounding of the data near the intersections of the stability curves comes about because in this region the system may become unstable by two processes, as described in the theory above. Overall the bridges are more sensitive to any external perturbations and to imperfections in the supporting disk edges to which the liquid surface is pinned. Thus, it becomes more difficult to move close to the cusp regions formed by the intersection of the stability curves.

The problem of stability in connected fluid domains is complex, and has many practical implications. Until now there had been no experimental verification of these issues to our knowledge. We have examined one of the simplest problems, viz., a pair of connected cylindrical bridges. Despite the small artifacts inherent in the magnetic levitation technique, it is clear that our experimental results confirm the theoretical predictions for connected cylindrical bridges. Although incremental improvements would be likely in a space-borne microgravity environment, our technique nevertheless allows us to examine both the stability and dynamics of connected domains in a wide variety of configurations. These will be the subject of future investigations.

This work was supported by the National Aeronautics and Space Administration's Microgravity Program under Grants No. NAG8-1779 and No. NAG3-1864.

[1] J. A. Plateau, *Statique Expérimentale et Théorique des Liquides Soumis aux Seules Forces Moléculaires* (Gautier-Villars, Paris, 1873).
[2] J. W. S. Rayleigh, Proc. R. Soc. London **29**, 71 (1879).
[3] A. Beer, Ann. Phys. Chem. **96**, 1 (1855); **96**, 210 (1855).
[4] G. Mason, J. Colloid Interface Sci. **32**, 172 (1970).
[5] S. R. Coriell, S. C. Hardy, and M. R. Cordes, J. Colloid Interface Sci. **60**, 126 (1977).
[6] J. Meseguer, J. Cryst. Growth **62**, 577 (1983).
[7] J. M. Vega and J. M. Perales, European Space Agency Publication Division Report No. ESA SP-191 1983 (unpublished), p. 247.
[8] A. D. Myshkis, V. G. Babskii, N. D. Kopachevskii, L. A. Slobozhanin, and A. D. Tyuptsov, *Low-Gravity Fluid Mechanics* (Springer-Verlag, Berlin, 1987).
[9] L. A. Slobozhanin and J. M. Perales, Phys. Fluids A **5**, 1305 (1993).
[10] N. A. Bezdenejnykh, J. Meseguer, and J. M. Perales, Phys. Fluids A **4**, 677 (1992).
[11] J. Meseguer, N. A. Bezdenejnykh, J. M. Perales, and P. Rodrígues de Francisco, Microgravity Sci. Technol. **VIII/1**, 2 (1995).
[12] A. Laverón-Simavilla and J. M. Perales, Phys. Fluids **7**, 1204 (1995).
[13] J. I. D. Alexander, S. Delafontaine, A. Resnick and W. C. Carter, Microgravity Sci. Technol. **IX/3**, 193 (1996).
[14] J. I. D. Alexander and L. A. Slobozhanin (unpublished).
[15] R. D. Gillette and D. C. Dyson, Arch. Ration. Mech. Anal. **53**, 150 (1974).
[16] V. R. Orel, J. Appl. Mech. Tech. Phys. **15**, 767 (1974).
[17] L. A. Slobozhanin, Fluid Dyn. **18**, 171 (1983).
[18] M. P. Mahajan, M. Tsige, P. L. Taylor, and C. Rosenblatt, Phys. Fluids **10**, 2208 (1998).
[19] M. P. Mahajan, M. Tsige, P. L. Taylor, and C. Rosenblatt, J. Colloid Interface Sci. **213**, 592 (1999).
[20] M. P. Mahajan, M. Tsige, S. Zhang, J. I. D. Alexander, P. L. Taylor, and C. Rosenblatt, Phys. Rev. Lett. **84**, 338 (2000).
[21] M. P. Mahajan, M. Tsige, S. Zhang, J. I. D. Alexander, P. L. Taylor, and C. Rosenblatt, Exp. Fluids (to be published).
[22] J. Meseguer, J. Fluid Mech. **130**, 123 (1983).
[23] J. Lowry and P. H. Steen, Proc. R. Soc. London, Ser. A **449**, 411 (1995).
[24] L. A. Slobozhanin, J. I. D. Alexander, and A. H. Resnick, Phys. Fluids **9**, 1893 (1997).